Why Mathematics?

Perhaps we can see more easily why one should study mathematics if we can take a moment to consider with mathematics is. Unfortunately the answer cannot be given in a single sentence or a single chapter. The subject has many facets or, some might say, is hydra-headed. One can look at mathematics as a language, as a particular kind of logical structure, as a body of knowledge about numbers in space, as a series of methods for driving conclusions, as the essence of our knowledge of the physical world, or merely as an amusing intellectual activity. Each of these features would in itself be difficult to describe it accurately in a brief space.

Because it is impossible to give a concise and readily understandable definition of mathematics, some writers of suggested, rather evasively, that mathematics is what mathematicians do. But mathematicians are human beings, and most of the things they do are uninteresting and some, embarrassing to relate. The only merit in this proposed definition of mathematics is that it points up to the fact that mathematics is a human creation.

A variation on the above definition which promises more help in understanding the nature, content, and the values in mathematics, is that mathematics is what *mathematics does.* If we examine mathematics from the standpoint of what it is intended to and does accomplish, we shall undoubtedly gain a truer and clearer picture of the subject.

Mathematics is concerned primarily with what can be accomplished by reasoning. And here we face the first hurdle. Why should one reason? It is not a natural activity for the human animal. It is clear that one does not need reasoning to learn how to eat or to discover what foods maintain life. Man knew how to feed, clothe, and house himself millenniums before mathematics existed. Getting along with the opposite sex is an art rather than a science mastered by reason. One can engage in a multitude of occupations and even climb high in the business and industrial world without much use of reasoning and certainty without mathematics. One's social position is hardly elevated by a display of his knowledge of trigonometry. In fact, civilizations in which reasoning and mathematics played no role have endured and even flourished. If one were willing to reason, he could readily supply evidence to prove that reasoning is a dispensable activity.

Those who are opposed to reasoning will read only point out other methods of obtaining knowledge. Most people are in fact convinced that their senses are really more than adequate. The very common assertion "seeing is believing" expresses the common reliance upon the senses. But everyone should recognize that the senses are limited and often fallible and, even more accurate, must be interpreted. Let us consider, as an example, the sense of sight. How big is the sun? Our eyes tell us that it is about as large as a rubber ball. This then is what we should believe. On the other

Adapted from Chapter 1 of *Mathematics for the Nonmathematician* by Morris Kline

From TOKTopics.com

hand, we do not see the air around us, nor for that matter can we feel, touch, smell, or taste it. Hence we should not believe in the existence of air.

To consider is somewhat more complicated situation, suppose a teacher should hold up a fountain pen and ask, what is it? A student coming from some primitive society may call it a shiny stick, and indeed this is what the eyes see. Those who call it a fountain pen are really calling upon education and experience stored in their minds. Likewise, when we look at the tall building from a distance, it is experience which tells us that the building is tall. Hence the old saying that "we are prone to see what lies behind our eyes, rather than what appears before then."

The senses are obviously helpless in obtaining some kinds of knowledge, such as the distance to the sun, the size of the earth, the speed of a bullet (unless one wishes to feel its velocity), the temperature of the sun, the production of eclipses, and dozens of other facts.

…

It may be clear from this brief discussion that the senses, measurement, and experimentation, to consider three alternative ways of acquiring knowledge, are by no means adequate in a variety of situations. Reasoning is essential. The lawyer, the doctor, the scientist, and the engineer all employ reasoning daily to derive knowledge that would otherwise not be obtainable or perhaps obtainable only at great expense and effort. Mathematics more than any other human endeavor relies upon reasoning to produce knowledge.

One may be willing to accept the fact that mathematical reasoning is an effective procedure. But just what does mathematics seem to accomplish with its reasoning? The primary objective of all mathematical work is to help manage study nature, and in this endeavor mathematics cooperates with science.

…

The fact that mathematics is of central importance in the study of nature reveals almost immediately several values of the subject. The first is the practical value. The construction of bridges and skyscrapers, the harnessing of the power of water, cold, electricity, and the atom, the effective employment of light, sound, and radio in illumination, communication, navigation, and even entertainment, and the advantageous employment of chemical knowledge in the design of materials, in the production of useful forms of oil, and medicine are but a few of the practical achievements already attained. And the future promises to dwarf the past.

However, material progress is not the most compelling reason for the study of nature, nor have practical results usually come about from investigations so directed. In fact, to overemphasize practical values is to lose sight of the greater significance of human thought. The deeper reason for the study of nature is to try to

understand the ways of nature, that is, to satisfy sheer intellectual curiosity. Indeed, to ask disinterested questions about nature is one of the distinguishing marks of mankind. In all civilizations some people at least have tried to answer such questions as: how did the universe come about? How old is the universe and the earth in particular? How large are the sun and the earth? Is man an accident or part of a larger design? Will the solar system continue to function or will the earth some day fall into the sun? What is light? Of course, not all people are interested in such questions. Food, shelter, sex, and television are enough to keep many happy. But others, aware of the pervasive natural mysteries, or more strongly obsessed to resolve them than any businessman is to acquire power and wealth.

Beyond the improvement in the material life of man and beyond satisfaction of intellectual curiosity, the study of nature offer is in tangible values of another sort, especially the abolition of fear and terror and their replacement by a deep, quiet satisfaction in the ways of nature. To the uneducated and to those uninitiated in the world of science, many manifestations of nature have appeared to be the agents of distraction sent by angry gods. Some of the believes in ancient and even medieval Europe may be of special interest in view of what happened later.

…

Fears, dread, and superstitions have been eliminated, at least in our Western civilization, by just those intellectually curious people who have studied nature's mighty displays. Those "seemingly unprofitable amusements a speculative brains" have freed us from serfdom, given us undreamed of powers, and, in fact, have replaced negative doctrines by positive mathematical laws which revealed a remarkable order and uniformity in nature. Man has emerged as the proud possessor of knowledge which has enabled him to view nature calmly and objectively. An eclipse of the sun occurring on schedule is no longer an occasion for trembling but for a quiet satisfaction that we know nature's ways. We breathe freely, knowing that nature will not be willful or capricious.

…

Practical, scientific, philosophical, and artistic problems have caused men to investigate mathematics. But there is one other motive which is as strong as any of these – the search for beauty. Mathematics is an art, and as such affords the pleasure which all the arts afford. This last statement may come as a shock to people who are used to the conventional concept of the true arts and mentally contrast these with mathematics to the detriment of the latter. But the average person has not thought through what the arts really are and what they offer. All that many people actually see in painting, for example, are familiar scenes and perhaps bright colors. These qualities, however, are not the ones which make painting an art. The real values must be learned, and a genuine appreciation of art calls for much study.

…

The appeal offered by the detachment and objectivity of mathematical reasoning is superbly described by Bertrand Russell.

> *Remote from human passions, remote even from the pitiful facts of nature, the generations have gradually created an ordered cosmos, where pure thought can dwell as in its natural home and where one, at least, of our nobler impulses can escape from the dreary exile of the actual world.*

The creation and contemplation of mathematics offer such values.

Despite all these arguments for the study of mathematics, the reader may have justifiable doubts. The idea that thinking about numbers and figures leads to deep and powerful conclusions which influence almost all other branches of thoughts may seem incredible.

…

Let us review our objectives. We should like to understand what mathematics is, how it functions, what it accomplishes for the world, and what it has to offer in itself. We hope to see that mathematics has content which serves the physical and social scientist, the philosopher, the logician, and the artists; content which influences the doctrines of the statesman and the theologian; content which satisfies the curiosity of the man who surveys the heavens and the man who muses on the sweetness of musical sounds; and content which has undeniably, if sometimes imperceptibly, shaped the course of modern history. In brief, we shall try to see that mathematics is an integral part of the modern world, one of the strongest forces shaping its thoughts and actions, and a body of living though inseparably connected with, dependent upon, and in turn valuable to all other branches of our culture. Perhaps we shall also see how by suffusing and influencing all thought it has set the intellectual temper of our times.